

Radio Frequency Carrier Arraying for Near Maximum Carrier Signal-to-Noise Ratio Improvement

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Radio frequency arraying of several receiving systems provides signal-to-noise ratio improvement relative to a single receiving system. This report considers radio frequency carrier arraying which provides near maximum signal-to-noise ratio improvement for carrier reception and demodulation.

I. Introduction

An earlier report ("Radio Frequency Carrier Arraying for High Rate Telemetry Reception," Ref. 1) presented a method for obtaining adequate radio frequency carrier signal-to-noise ratio improvement for high-rate telemetry reception by arraying receiving systems or stations. This report considers RF carrier arraying of receiving systems or stations to provide near maximum improvement in carrier signal-to-noise ratio for coherent RF carrier reception and RF carrier demodulation which is applicable to low-rate telemetry with its attendant smaller RF carrier margin. This RF carrier signal-to-noise ratio improvement (achieved by arraying) provides a corresponding improvement in minimum RF carrier level for radio metric tracking (e.g., doppler and ranging) relative to a single receiving system or station.

added to expand the signal-to-noise improvement capability for RF carrier arraying. In addition, each receiving system can be fed from its own antenna. The concept shown in Fig. 1 is similar to that presented for high-rate telemetry reception (Ref. 1), with some additions to the block diagram which will be discussed later in this report. The following discussion of the input signal(s), which was presented in Ref. 1, is included here for clarification. The input signal to N receiving systems or stations is a radio frequency signal $(\cos \omega_{RF}t)$ phase-modulated with telemetry and/or a ranging waveform. Consider, as an example, the case for telemetry where the RF carrier is phase-modulated with a square-wave subcarrier $(\cos \omega_{sc}t)$ at a peak modulation index m_{pD} that is in turn, biphasemodulated with data $D(t)$. Refer to Fig. 1 during the following development. The signal at the input to the low noise amplifier in receiving system 1 is

II. Receiver Configuration

Figure 1 illustrates the method for achieving RF carrier arraying for near maximum carrier signal-to-noise ratio improvement. Two receiving systems are shown to illustrate the concept. However, additional receiving systems can be

$$2^{1/2} A_1(t) \cos$$

$$[\omega_{RF}t + \theta_{i1} + D(t + \tau_{i1}) \cdot m_{pD} \cdot \cos(\omega_{sc}t + \theta_{sc_{i1}})]$$

$$+ n_{i1}(t) \quad (1a)$$

For a binary modulation waveform, the carrier component becomes

$$2^{1/2} A_1(t) \cdot \cos m_{pD} \cos(\omega_{RF} t + \theta_{i1})$$

The sideband component becomes

$$+ 2^{1/2} A_1(t) \sin m_{pD} \cdot D(t + \tau_{i1}) \cos(\omega_{sc} t + \theta_{sc i1}) \cdot \sin(\omega_{RF} t + \theta_{i1}) \quad (1b)$$

and the noise component is $n_{i1}(t)$.

For receiving system 2, the corresponding input signal is

$$2^{1/2} A_2(t) \cdot \cos m_{pD} \cos(\omega_{RF} t + \theta_{i2} + 2^{1/2} A_2(t) \cdot \sin m_{pD} \cdot D(t + \tau_{i2}) \cdot \cos(\omega_{sc} t + \theta_{sc i2}) \cdot \sin(\omega_{RF} t + \theta_{i2}) + n_{i2}(t). \quad (2)$$

The other $N-2$ receiving systems will have corresponding input signals. Since antenna 1 and antenna 2 are separate antennas, physically separated, the phase shift on the RF carrier (θ_{i2}) and square wave subcarrier ($\theta_{sc i2}$) and the group delay on the data (τ_{i2}) in receiving system 2 are different from the corresponding phase shifts and modulation group delay in receiving system 1. In general, the differences in RF carrier phase shift ($\theta_{i2} - \theta_{i1}$), subcarrier phase shift ($\theta_{sc i2} - \theta_{sc i1}$) and modulation group delay ($\tau_{i2} - \tau_{i1}$) will vary with time during a station pass. The above applies in general to all N receiving systems. The terms $n_{i1}(t)$, $n_{i2}(t)$, etc. represent a combination of galactic noise, atmospheric noise, noise in the antenna sidelobes due to the Earth, noise due to losses in microwave reflectors, and noise due to losses in microwave components all lumped with noise due to input amplifier(s). This combined noise is measured relative to reference temperature load(s) connected to the amplifier input (during the measurement) and designated as the operating equivalent system temperature T_{op1} , T_{op2} , etc. The noise term $n_{i1}(t)$ has a double-sided noise spectral density $N_{01}/2 = (k \cdot T_{op1} \cdot 1)/2$ watts/Hz, where k is Boltzmann's constant, $1.38 \cdot 10^{-23}$ joule/K. The noise terms $n_{i2}(t)$, etc. have noise spectral densities related to T_{op2} , etc. as above.

As described in Ref. 1, the first local oscillator signal, which is derived from a voltage-controlled oscillator (VCO) through a frequency multiplier (XM), is:

$$2^{1/2} \cos(\omega_{LO} t + \hat{\theta}_{RF}) \quad (3)$$

at the first mixer in receiving system 1. The term $\hat{\theta}_{RF}$ represents a noisy estimate of the input signal phase to the first mixer. Note that this first local oscillator signal (expression 3) is also applied to the first mixers of the other receiving systems. In particular, this local oscillator signal will experience an additional phase shift of $Y_2 \cdot 2\pi + \theta_2$ to the first mixer of receiving system 2 where Y_2 is an integer (a large integer for separate antennas). As a consequence, the first local oscillator signal (receiving system 2) becomes:

$$2^{1/2} \cos(\omega_{LO} t + \hat{\theta}_{RF} + \theta_2) \quad (4)$$

For receiving system N , the additional first local oscillator phase shift is $Y_N \cdot 2\pi + \theta_N$, which provides a local oscillator signal $2^{1/2} \cos(\omega_{LO} t + \hat{\theta}_{RF} + \theta_N)$ to its first mixer.

Description of the received signal as it passes through receiving system 1 was presented in Ref. 1. At the output of second IF filter F_{A1} (which is narrow band relative to the sidebands) the signal becomes:

$$K_{A1} \cdot K_{DIST1} \cdot 2^{1/2} A_1 \cos m_{pD} \cdot \cos [\omega_{REF2} t + (\theta_{FA10} - \hat{\theta}_{RF})] + n_{FA10}(t) \quad (5)$$

where θ_{FA10} represents the RF phase shift up to this point in system 1 and $n_{FA10}(t)$ represents the amplified receiver noise related to T_{op1} in the noise bandwidth of F_{A1} . Note that since the gain from (5) in Fig. 1 to the AGC detector is constant at K_{2ndIF1} (minus the summing junction loss), the attenuator ($Atten_1$) at the input to the second IF amplifier in receiving system 1 provides a means for setting the signal level to provide the required output level for the telemetry subcarrier spectrum in receiving system 1 when RF carrier arraying with receiving system 2, etc. This attenuator setting ($Atten_1$) also establishes the proper level for the ranging spectrum at the output of the second IF distribution amplifier in receiving system 1 for ranging demodulation (not shown).

Description of the received signal as it passes through receiving system 2 was also presented in Ref. 1. Note that the receiving systems considered here are similar so that the signal-to-noise ratio at the output of the first IF amplifier (with gain K_{IFV2}) in receiving system 2 is nearly equal to that at the corresponding point in receiving system 1. (The concept developed herein however, can be applied to arraying receiving systems where the signal-to-noise ratios are not equal.) The second local oscillator signal in receiving system 2 is derived

from a voltage-controlled crystal oscillator (VCO_2) through a frequency multiplier (XQ). From inspection of Fig. 1, the frequency $VCO_2 \cdot XQ$ is equal to $(\omega_{REF_1} t)$ although its phase is modified by differences in carrier phase as pointed out in the discussion relative to expression (2) plus any differences in carrier phase relative to receiving system 1 up to this point in system 2. The second local oscillator signal can be expressed as

$$2^{1/2} \cos (\omega_{REF_1} t - \hat{\theta}_{LO_2}) \quad (6)$$

where $\hat{\theta}_{LO_2}$ represents the estimate of the RF carrier input phase to the lower second mixer (receiving system 2). The phase estimate $\hat{\theta}_{LO_2}$ is derived from a phase locked loop whose closed loop noise bandwidth is a small fraction of the closed loop noise bandwidth of the RF carrier phase tracking loop in receiving system 1 which provides the first local oscillator signal. Hence the noise on phase estimate $\hat{\theta}_{LO_2}$ is much less than that on $\hat{\theta}_{RF}$ (first local oscillator signal). Note that if the carrier signal-to-noise ratio in receiving system 2 were significantly less than that in receiving system 1, this effect could be offset by making the closed loop noise bandwidth in receiving system 2 a still smaller fraction of that in receiving system 1.

Although the noise on phase estimate $\hat{\theta}_{LO_2}$ is much less than that on $\hat{\theta}_{RF}$, the effect of the noise on phase estimates $\hat{\theta}_{LO_2}, \hat{\theta}_{LO_3}, \dots, \hat{\theta}_{LO_N}$ is cumulative (in an rms sense) in the summing junction when arraying many receiving systems. This cumulative phase noise effect is kept small relative to the phase noise on $\hat{\theta}_{RF}$ (the first local oscillator) by including an additional second mixer (upper mixer) shown in receiving system 2 with the second local oscillator fed to this additional second mixer through a narrow-band phase locked local oscillator tracking loop. The phase noise on the output of the local oscillator tracking loop $\hat{\theta}'_{LO_2}$ is small compared to the phase noise at its input. Consequently, the second local oscillator signal fed to the additional (upper) second mixer becomes

$$2^{1/2} \cos (\omega_{REF_1} t - \hat{\theta}'_{LO_2}) \quad (7)$$

Receiving system 2 shows two second IF distribution amplifiers (see Fig. 1). The upper IF distribution amplifier accepts the output of the upper second mixer (described above) and in turn provides its output signal to the upper IF filter F_{A_2} (and to the telemetry IF channel) in receiving system 2.

Note that the additional filtering of phase estimate $\hat{\theta}_{LO_2}$ and the additional (upper) second mixer, IF distribution amplifier and IF filter F_{A_2} described in the preceding paragraph represent additions to Fig. 1 relative to Ref. 1. These additions to receiving systems 2 through N provide the near maximum

signal-to-noise ratio improvement for RF carrier arraying presented in this report.

The upper and lower second IF filter F_{A_2} in receiving system 2 have the same noise bandwidth as IF filter F_{A_1} in receiving system 1 (by design). The signal at the output of the upper IF filter F_{A_2} (with gain K_{A_2}) can be expressed as

$$K_{A_2} \cdot K_{DIST_2} \cdot 2^{1/2} A_2 \cos m_{PD} \cdot \cos \left[\omega_{REF_2} t + \left(\theta_{FA_{20}} - \left(\hat{\theta}_{RF} - \hat{\theta}'_{LO_2} \right) \right) \right] + n_{FA_{20}}(t) \quad (8)$$

where $\theta_{FA_{20}}$ represents the RF phase shift up to this point in system 2 and $n_{FA_{20}}(t)$ represents the amplified receiver noise in the noise bandwidth of F_{A_2} at the operating equivalent system noise temperature T_{OP_2} . This signal is provided as an input to the summing junction (see Fig. 1). The signal at the output of the lower IF filter F_{A_2} is the same as expression (8), with $\hat{\theta}'_{LO_2}$ replaced by $\hat{\theta}_{LO_2}$, which has higher phase noise as discussed in the preceding paragraph. Note that the phase shift from the output of the upper IF filter F_{A_2} (expression 8) in Fig. 1 to the phase detector which provides the error signal to the tracking filter (receiving system 2) is a constant by design. Consequently the phase shifter marked A , which is in series with the input (ω_{REF_2}) in receiving system 2 provides a means for setting the RF phase of expression (8) equal to the RF phase of expression (5) in receiving system 1 at the summing junction. The second IF amplifier gain K_{2ndIF_2} is designed so that the signal level $(K_{A_2} K_{DIST_2} 2^{1/2} A_2)$ can be set as required relative to the signal level $(K_{A_1} \cdot K_{DIST_1} \cdot 2^{1/2} A_1)$ in receiving system 1 at the input to the summing junction.

III. Predetection Signal-to-Noise Ratio and Resultant Phase Noise

As developed in Ref. 1, with the other receiving system(s) (2 through N) switched out of the summing junction, the predetection carrier signal-to-noise power ratio in receiving system 1 represented by expression (5) is

$$P_{c_1} / P_{n_1} = \frac{A_1^2 \cos^2 m_{PD}}{NBW_{FA_1} \cdot N_{0_1}} \text{ or } \frac{P_{c_1}}{NBW_{FA_1} \cdot N_{0_1}} \quad (9)$$

where NBW_{FA_1} represents the noise bandwidth of F_{A_1} and N_{0_1} is the one-sided noise spectral density for receiving system 1 which was defined in the discussion relative to expression (1). The receiving system contains a second-order RF carrier phase tracking loop which utilizes a bandpass limiter and a

sinusoidal phase detector. Utilizing the information in Refs. 2 and 3 and limiting $\sigma_n \leq 1$ radian, the rms phase noise σ_{ϕ_n} at the output of the RF carrier tracking loop (i.e., on the first local oscillator signal) can be expressed as:

$$\sigma_{\phi_{n1}} = \frac{\frac{N_{01}}{2} \cdot 2B_{L1}}{P_{c1}} \left[\frac{1 + \frac{P_{c1}}{NBW_{FA1} \cdot N_{01}}}{0.862 + \frac{P_{c1}}{NBW_{FA1} \cdot N_{01}}} \cdot \frac{\exp\left(\frac{N_{01} B_{L1}}{P_{c1}}\right)}{\sinh\left(\frac{N_{01} B_{L1}}{P_{c1}}\right)} \right]^{1/2} \text{ radians, rms} \quad (10)$$

where $2B_{L1}$ is the two-sided closed loop noise bandwidth of the RF carrier phase tracking loop. As described in Ref. 1,

$$2B_{L1} = \frac{2B_{LO1}}{r_0 + 1} \cdot 1 + \left(r_0 \frac{\alpha}{\alpha_{01}} \right)$$

where $r_0 = 2$ by design (0.707 damping) and $2B_{LO1}$ is the design point (threshold) two-sided closed loop noise bandwidth in receiving system 1. The term α_1 is the limiter suppression factor resulting from the noise power to carrier power ratio in NBW_{FA1} , α_1 has a value of α_{01} at design point (threshold). At threshold, the predetection carrier signal-to-noise ratio in a noise bandwidth equal to the threshold closed loop noise bandwidth ($2B_{LO}$) is unity (i.e., $P_c/2B_{LO} \cdot N_0 = 1$).

With receiving system 2 connected to the summing junction, the summed predetection carrier signal-to-noise ratio in receiving system 1 (as developed in Ref. 1) becomes:

$$\frac{P_{c1 \Sigma 1,2}}{P_{n1 \Sigma 1,2}} = \frac{P_{c1}}{NBW_{FA1} \cdot N_{01}} \cdot \frac{(1 + \beta_2 \gamma_2)^2}{1 + \beta_2^2} \quad (11)$$

where β_2 is the voltage coupling of receiving system 2 relative to receiving system 1 into the summing junction and γ_2^2 is the carrier power-to-noise spectral density of the receiving system 2 relative to the receiving system 1. The resultant phase noise on the first local oscillator due to this summed predetection carrier signal-to-noise ratio is (Ref. 1)

$$\sigma_{\phi_{n1 \Sigma 1,2}} = \frac{\frac{N_{01}}{2} \cdot 2B_{L1}}{P_{c1}} \cdot \frac{1}{\eta_2} \left[\frac{1 + \frac{P_{c1} \eta_2}{NBW_{FA1} \cdot N_{01}}}{0.862 + \frac{P_{c1} \eta_2}{NBW_{FA1} \cdot N_{01}}} \cdot \frac{\exp\left(\frac{N_{01} B_{L1}}{P_{c1} \eta_2}\right)}{\sinh\left(\frac{N_{01} B_{L1}}{P_{c1} \eta_2}\right)} \right]^{1/2} \quad (12)$$

where $\eta_2 = (1 + \beta_2 \gamma_2)^2 / (1 + \beta_2^2)$ for two receiving systems arrayed. The corresponding $\sigma_{\phi_{n1 \Sigma 1, \dots, N}}$ for N similar receiving systems arrayed become

$$\sigma_{\phi_{n1 \Sigma 1, \dots, N}} = \frac{\frac{N_{01}}{2} \cdot 2B_{L1}}{P_{c1}} \cdot \frac{1}{\eta_N} \left[\frac{1 + \frac{P_{c1} \cdot \eta_N}{NBW_{FA1} \cdot N_{01}}}{0.862 + \frac{P_{c1} \cdot \eta_N}{NBW_{FA1} \cdot N_{01}}} \cdot \frac{\exp\left(\frac{N_{01} \cdot B_{L1}}{P_{c1} \cdot \eta_N}\right)}{\sinh\left(\frac{N_{01} \cdot B_{L1}}{P_{c1} \cdot \eta_N}\right)} \right]^{1/2} \quad (13)$$

where $\eta_N = (1 + \beta_2 \gamma_2 + \dots + \beta_N \gamma_N)^2 / (1 + \beta_2^2 + \dots + \beta_N^2)$ for N receiving systems arrayed.

Up to this point, first local oscillator phase noise due to predetection carrier signal-to-noise ratio in receiving system 1 has been examined. However, phase noise on the first local oscillator is due not only to the predetection carrier signal-to-noise ratio as described above; it also includes the local oscillator (VCO_2 XQ) phase noise from receiving system 2 which is coupled through the summing junction into the phase locked loop in receiving system 1 (Ref. 1). The rms phase noise $\sigma_{\phi_{n2}}$ on phase estimate $\hat{\theta}_{LO2}$ is derived with a two-sided closed loop noise bandwidth $2B_{L2}$ in receiving system 2 (see expression 17 below). The additional filtering provided by the local oscillator tracking loop in receiving system 2, described relative to expression (7) above, reduces the rms phase noise $\sigma_{\phi_{n2}}$

to a value $\sigma'_{\phi_{n2}}$. This output rms phase noise $\sigma'_{\phi_{n2}}$ is less than $\sigma_{\phi_{n2}}$ by the square root of the ratio of the local oscillator tracking loop noise bandwidth to $2B_{L2}$. Consequently the total rms phase noise on the first local oscillator (Ref. 1)

$$\left[\sigma_{\phi_{n1\Sigma,2}}^2 + \left(\frac{\beta_2 \sigma'_{\phi_{n2}}}{1 + \beta_2} \right)^2 \right]^{1/2} \quad (14)$$

is essentially equal to $\sigma_{\phi_{n1\Sigma,2}}^2$. From inspection of expressions (11) and (12), the improvement in carrier signal-to-noise ratio for two receiving systems arrayed approaches $(1 + \beta_2 \gamma_2)^2 / (1 + \beta_2^2)$. Note that for N receiving systems arrayed, the total rms phase noise on the first local oscillator is

$$\left[\sigma_{\phi_{n1\Sigma 1, \dots, N}}^2 + \left(\frac{\beta_2 \sigma'_{\phi_{n2}}}{1 + \beta_2} \right)^2 + \dots + \left(\frac{\beta_N \sigma'_{\phi_{nN}}}{1 + \beta_N} \right)^2 \right]^{1/2} \quad (15)$$

The total rms phase noise represented by expression (15) can be considered as due to an equivalent predetection carrier signal-to-noise ratio for N receiving systems arrayed. To the extent that expression (15) is essentially equal to $\sigma_{\phi_{n1\Sigma 1, \dots, N}}$, the improvement in carrier signal-to-noise ratio approaches $(1 + \beta_2 \gamma_2 + \dots + \beta_N \gamma_N)^2 / (1 + \beta_2^2 + \dots + \beta_N^2)$. In any case, comparison of this equivalent predetection carrier signal-to-noise ratio with the initial predetection carrier signal-to-noise ratio in a single receiving system (i.e., system 1) provides the improvement due to radio frequency carrier arraying for N receiving systems.

As developed in Ref. 1, the predetection carrier signal-to-noise power ratio in receiving system 2 represented by expression (8) is:

$$\frac{P_{c2\Sigma 1,2}}{P_{n2}} = \frac{A_2 \cos^2 m_{pD} \left(1 - \frac{\sigma_{\phi_{n1\Sigma 1,2}}^2}{2} \right)^2}{NBW_{FA2} \cdot N_{02}}$$

or

$$\frac{P_{c2} \left(1 - \frac{\sigma_{\phi_{n1\Sigma 1,2}}^2}{2} \right)^2}{NBW_{FA2} \cdot N_{02}} \quad (16)$$

The carrier tracking loop in receiving system 2 is a second-order loop ($r_0 = 2$) which also utilizes a bandpass limiter and a sinusoidal phase detector. The rms phase noise $\sigma_{\phi_{n2}}$ in receiving system 2 carrier tracking loop becomes

$$\sigma_{\phi_{n2\Sigma 1,2}} = \frac{\frac{N_{02}}{2} 2B_{L2}}{P_{c2\Sigma 1,2}}$$

$$\left[\frac{1 + \frac{P_{c2\Sigma 1,2}}{NBW_{FA2} \cdot N_{02}} \exp \left(\frac{N_{02} \cdot B_{L2}}{P_{c2\Sigma 1,2}} \right)}{0.862 + \frac{P_{c2\Sigma 1,2}}{NBW_{FA2} \cdot N_{02}} \sinh \left(\frac{N_{02} B_{L2}}{P_{c2\Sigma 1,2}} \right)} \right]^{1/2} \quad (17)$$

radians, rms

where

$$P_{c2\Sigma 1,2} = P_{c2} \left(1 - \frac{\sigma_{\phi_{n1\Sigma 1,2}}^2}{2} \right)^2$$

for two receiving systems arrayed.

For N receiving systems arrayed, the predetection carrier signal-to-noise power ratio in receiving system 2 becomes

$$\frac{P_{c2\Sigma 1, \dots, N}}{P_{n2}} = \frac{P_{c2} \left(1 - \frac{\sigma_{\phi_{n1\Sigma 1, \dots, N}}^2}{2} \right)^2}{NBW_{FA2} \cdot N_{02}} \quad (18)$$

and the corresponding $\sigma_{\phi_{n2\Sigma 1, \dots, N}}$ can be determined by substitution of $P_{c2\Sigma 1, \dots, N}$ in the place of $P_{c2\Sigma 1,2}$ in expression (17). The predetection carrier signal-to-noise power ratio in receiving system N becomes

$$\frac{P_{cN\Sigma 1, \dots, N}}{P_{nN}} = \frac{P_{cN} \left(1 - \frac{\sigma_{\phi_{n1\Sigma 1, \dots, N}}^2}{2} \right)^2}{NBW_{FAN} \cdot N_{0N}} \quad (19)$$

The resultant rms phase noise $\sigma_{\phi_{nN}}$ in receiving system N carrier tracking loop is

$$\sigma_{\phi_{nN \Sigma 1, \dots, N}} = \frac{\frac{N_{0N}}{2} \cdot 2B_{LN}}{P_{c_{N \Sigma 1, \dots, N}}} \left[\frac{1 + \frac{P_{c_{N \Sigma 1, \dots, N}}}{NBW_{F_{AN}} \cdot N_{0N}}}{0.862 + \frac{P_{c_{N \Sigma 1, \dots, N}}}{NBW_{F_{AN}} \cdot N_{0N}}} \right] \cdot \frac{\exp\left(\frac{N_{0N} \cdot B_{LN}}{P_{c_{N \Sigma 1, \dots, N}}}\right)}{\sinh\left(\frac{N_{0N} \cdot B_{LN}}{P_{c_{N \Sigma 1, \dots, N}}}\right)} \Bigg]^{1/2} \text{ radians, rms} \quad (20)$$

IV. Performance

The improvement in carrier signal-to-noise ratio resulting from arraying receiving systems as shown in Fig. 1 can be determined using the preceding development. In order to illustrate performance improvement, a representative set of design parameters are used to provide the performance shown in Figs. 2 and 3. The following parameters are used for receiving system 1:

Threshold two-sided noise bandwidth ($2B_{LO1}$) = 12 Hz.

Predetection IF filter noise bandwidth ($NBW_{F_{A1}}$) = 2200 Hz.

The parameters used for receiving system 2 through N are:

Threshold two-sided noise bandwidth ($2B_{LO2, \dots, N}$) = 1 Hz.

Predetection IF filter noise bandwidth ($NBW_{F_{A2, \dots, N}}$) = 2200 Hz.

Local oscillator tracking loop two-sided noise bandwidth = 0.1 Hz.

It is interesting to note that essentially the same performance as shown in Figs. 2 and 3 is obtained with $2B_{LO2, \dots, N}$ equal to 0.1 Hz (and local oscillator tracking loop bandwidth equal to 0.1 Hz).

Figure 2 shows the near maximum RF carrier signal-to-noise ratio improvement which results from arraying up to 12 similar receiving systems. Performance is shown for $\gamma_N = \beta_N$ for values of 1.0, 0.95, 0.90, and 0.84, where γ_N and β_N are defined in the preceding development and on Fig. 2. Note that for 10 receiving systems arrayed, the improvement in carrier signal-to-noise ratio for $\gamma_N = \beta_N = 1$ is 9.6 dB compared to maximum improvement which is

$$\frac{(1 + \beta_2 \gamma_2 + \dots + \beta_{10} \gamma_{10})^2}{1 + \beta_2^2 + \dots + \beta_{10}^2} (= 10) \text{ or } 10 \text{ dB,}$$

Figure 3 shows the effect of varying β_N , the voltage coupling of receiving system N into the summing junction relative to receiving system 1, on carrier signal-to-noise ratio improvement for 2, 3, 4, and 10 systems arrayed. Note that voltage coupling can vary considerably with a small resultant change in performance.

References

1. Brockman, M. H., "Radio Frequency Carrier Arraying for High Rate Telemetry," in *The Deep Space Network Progress Report 42-45*, pp. 209-223, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1978.
2. Tausworthe, R. C., *Theory and Practical Design of Phase-Locked Receivers*, Vol. I., Technical Report 32-819, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1966.
3. Tausworthe, R. C., "Limiters in Phase-Locked Loops: A Correction to Previous Theory," in *Space Program Summary No. 37-54, Vol. III*, pp. 201-203, Jet Propulsion Laboratory, Pasadena, Calif., 1968.

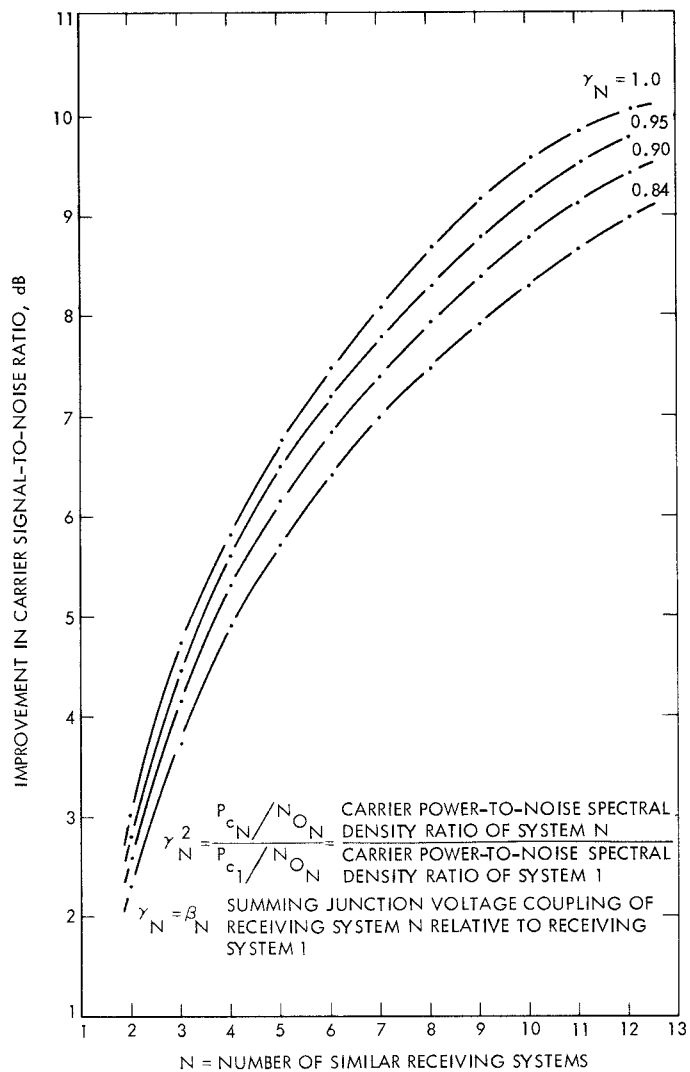


Fig. 2. Near maximum carrier signal-to-noise ratio improvement for radio frequency carrier arraying for reception vs number of receiving systems

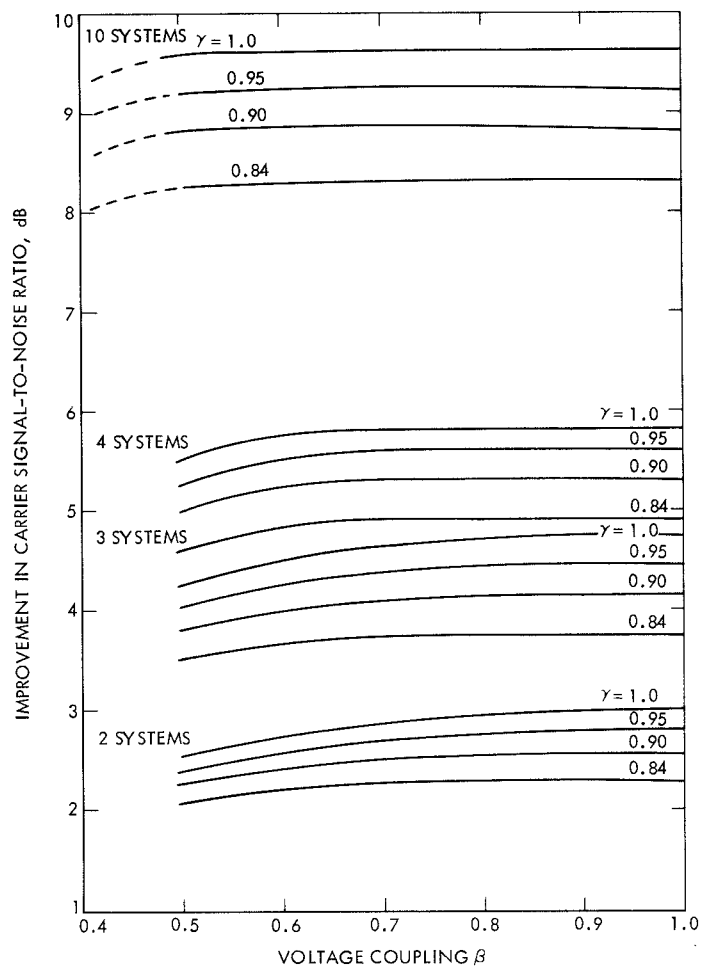


Fig. 3. Effect of summing junction voltage coupling on near maximum carrier signal-to-noise ratio improvement